NCERT Solutions Class 12 Maths Chapter 2
Exercise 2.2

Question 1:

Prove $3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

Solution:

Let $x = \sin\theta$ $x = \sin\theta$

Hence, $\sin^{-1}(x) = \theta$ $\sin^{-1}(x) = \theta$

Now,

$$\begin{aligned} RHS &= \sin^{-1}\left(3x - 4x^3\right) \\ &= \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right) \\ &= \sin^{-1}(\sin 3\theta) \\ &= 3\theta \\ &= 3\sin^{-1}x \\ &= LHS \end{aligned}$$

Question 2:

Prove $3\cos^{-1}x = \cos^{-1}\left(4x^3 - 3x\right), x \in \left[\frac{1}{2}, 1\right]$.

Solution:

Let $x = \cos\theta$

Hence, $\cos^{-1}(x) = \theta$

Now,

$$\begin{aligned} RHS &= \cos^{-1}\left(4x^3 - 3x\right) \\ &= \cos^{-1}\left(4\cos^3\theta - 3\cos\theta\right) \\ &= \cos^{-1}(\cos 3\theta) \\ &= 3\theta \\ &= 3\cos^{-1}x \\ &= LHS \end{aligned}$$

Question 3:

Prove $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$.

Solution:

Since we know that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$

Now,

$$LHS=\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}$$
$$=\tan^{-1}\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\cdot\frac{7}{24}}$$
$$=\tan^{-1}\left(\frac{\frac{48+77}{264}}{\frac{264-14}{264}}\right)$$
$$=\tan^{-1}\left(\frac{125}{250}\right)$$
$$=\tan^{-1}\left(\frac{1}{2}\right)$$
$$=RHS$$

Question 4:

Prove $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$.

Solution:

Since we know that $2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}$ and $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$

Now,

$$LHS = 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{28+3}{21}}{\frac{21-4}{21}}\right)$$

$$= \tan^{-1}\left(\frac{31}{17}\right)$$

$$= RHS$$

Question 5:

Write the function in the simplest form: $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}, x \neq 0$

Solution:

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

Hence,

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$=\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$=\tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$=\frac{\theta}{2}$$

$$=\frac{1}{2}\tan^{-1}x$$

Question 6:

Write the function in the simplest form: $\tan^{-1}\frac{1}{\sqrt{x^2-1}},|x|>1$

Solution:

Let $x=\text{cosec}\theta \Rightarrow \theta=\text{cosec}^{-1}x$

Hence,

$$\tan^{-1}\frac{1}{\sqrt{x^2-1}}=\tan^{-1}\frac{1}{\sqrt{\text{cosec}^2\theta-1}}$$

$$=\tan^{-1}\left(\frac{1}{\cot\theta}\right)$$

$$=\tan^{-1}(\tan\theta)$$

$$=\theta$$

$$=\text{cosec}^{-1}x$$

$$=\frac{\pi}{2}-\sec^{-1}x$$

Question 7:

Write the function in the simplest form: $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right),0<x<\pi$

Solution:

Since, $1-\cos x=2\sin^2\frac{x}{2}$ and $1+\cos x=2\cos^2\frac{x}{2}$

Hence,

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)=\tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)$$

$$=\tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$$

$$=\tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$=\frac{x}{2}$$

Question 8:

Write the function in the simplest form: $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), 0<x<\pi$

Solution:

$$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)=\tan^{-1}\left(\frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\cos x+\sin x}{\cos x}}\right)$$

$$=\tan^{-1}\left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right)$$

$$=\tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$=\tan^{-1}(1)-\tan^{-1}(\tan x)$$

$$=\frac{\pi}{4}-x$$

Question 9:

Write the function in the simplest form: $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}, |x|<a$

Solution:

Let $x = a\sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{x}{a}\right)$

Hence,

$$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} = \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$$
$$= \tan^{-1}\left(\frac{a\sin\theta}{a\sqrt{1-\sin^2\theta}}\right)$$
$$= \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$
$$= \tan^{-1}(\tan\theta)$$
$$= \theta$$
$$= \sin^{-1}\frac{x}{a}$$

Question 10:

Write the function in the simplest form: $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right), a>0; \frac{-a}{\sqrt{3}} \le x \le \frac{a}{\sqrt{3}}$

Solution:

Let $x = a\tan\theta \Rightarrow \theta = \tan^{-1}\left(\frac{x}{a}\right)$

Hence,

$$\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right) = \tan^{-1}\left(\frac{3a^2\cdot a\tan\theta - a^3\tan^3\theta}{a^3-3a.a^2\tan^2\theta}\right)$$
$$= \tan^{-1}\left(\frac{3a^3\tan\theta - a^3\tan^3\theta}{a^3-3a^3\tan^2\theta}\right)$$
$$= \tan^{-1}(\tan 3\theta)$$
$$= 3\theta$$
$$= 3\tan^{-1}\frac{x}{a}$$

Question 11:

Write the function in the simplest form: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

Solution:

Let $\sin^{-1}\frac{1}{2}=x$

Hence,

$$\sin x=\frac{1}{2}$$
$$=\sin\left(\frac{\pi}{6}\right)$$
$$x=\frac{\pi}{6}$$
$$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$$

Therefore,

$$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]=\tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$
$$=\tan^{-1}\left[2\cos\frac{\pi}{3}\right]$$
$$=\tan^{-1}\left[2\times\frac{1}{2}\right]$$
$$=\tan^{-1}[1]$$
$$=\frac{\pi}{4}$$

Question 12:

Find the value of $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$

Solution:

Since $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$

Hence,

$$\cot\left(\tan^{-1}a+\cot^{-1}a\right)=\cot\left(\frac{\pi}{2}\right)$$
$$=0$$

Question 13:

Find the value of $\tan\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right), |x|<1, y>0$ and $xy<1$.

Solution:

Let $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

Hence,

$$\begin{aligned}\sin^{-1}\frac{2x}{1+x^2} &= \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\\ &= \sin^{-1}(\sin 2\theta)\\ &= 2\theta\\ &= 2\tan^{-1}x\end{aligned}$$

Now, let $y=\tan\phi \Rightarrow \phi=\tan^{-1}y$

Hence,

$$\begin{aligned}\cos^{-1}\frac{1-y^2}{1+y^2} &= \cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right)\\ &= \cos^{-1}(\cos 2\phi)\\ &= 2\phi\\ &= 2\tan^{-1}y\end{aligned}$$

Therefore,

$$\begin{aligned}\tan\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right) &= \tan\frac{1}{2}\left(2\tan^{-1}x+2\tan^{-1}y\right)\\ &= \tan\left(\tan^{-1}x+\tan^{-1}y\right)\\ &= \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]\\ &= \left(\frac{x+y}{1-xy}\right)\end{aligned}$$

Question 14:

If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, find the value of x.

Solution:

It is given that $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$

Since we know that $\sin(x+y)=\sin x\cos y+\cos x\sin y$
Therefore,

$$\sin\left(\sin^{-1}\frac{1}{5}\right)\cos\left(\cos^{-1}x\right)+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$$

$$\left(\frac{1}{5}\right)\times(x)+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$$

$$\frac{x}{5}+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1 \qquad \ldots(1)$$

Now, let $\sin^{-1}\frac{1}{5}=y\Rightarrow \sin y=\frac{1}{5}$

Then,

$$\cos y=\sqrt{1-\left(\frac{1}{5}\right)^2}$$

$$=\frac{2\sqrt{6}}{5}$$

$$y=\cos^{-1}\left(\frac{2\sqrt{6}}{5}\right)$$

Therefore,

$$\sin^{-1}\frac{1}{5}=\cos^{-1}\left(\frac{2\sqrt{6}}{5}\right) \qquad \ldots(2)$$

Now, let $\cos^{-1}x=z\Rightarrow \cos z=x$
Then,

$$\sin z=\sqrt{1-x^2}$$

$$z=\sin^{-1}\sqrt{1-x^2}$$

Therefore,

$$\cos^{-1}x=\sin^{-1}\sqrt{1-x^2} \qquad \ldots(3)$$

From (1),(2) and (3), we have

$$\Rightarrow \frac{x}{5} + \cos\left(\cos^{-1}\frac{2\sqrt{6}}{5}\right)\sin\left(\sin^{-1}\sqrt{1-x^2}\right) = 1$$

$$\Rightarrow \frac{x}{5} + \frac{2\sqrt{6}}{5}\sqrt{1-x^2} = 1$$

$$\Rightarrow x + 2\sqrt{6}\sqrt{1-x^2} = 5$$

$$\Rightarrow 5 - x = 2\sqrt{6}\sqrt{1-x^2}$$

On squaring both the sides

$$25 + x^2 - 10x = 24 - 24x^2$$

$$25x^2 - 10x + 1 = 0$$

$$(5x-1)^2 = 0$$

$$(5x-1) = 0$$

$$x = \frac{1}{5}$$

Question 15:

If $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x-1} = \frac{\pi}{4}$, find the value of x.

Solution:

It is given that $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x-1} = \frac{\pi}{4}$

Since $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Therefore,

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{(x-1)(x+2)+(x+1)(x-2)}{(x+2)(x-2)-(x-1)(x+1)}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{2x^2-4}{-3}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan\left[\tan^{-1}\frac{4-2x^2}{3}\right]=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{4-2x^2}{3}=1$$

$$\Rightarrow 4-2x^2=3$$

$$\Rightarrow 2x^2=1$$

$$\Rightarrow x^2=\frac{1}{2}$$

$$\Rightarrow x=\pm\frac{1}{\sqrt{2}}$$

Question 16:

Find the value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

Solution:

Since, $\sin\theta = \sin(\pi-\theta)$

Therefore,

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left[\sin\left(\pi-\frac{2\pi}{3}\right)\right]$$

$$=\sin^{-1}\left(\sin\frac{\pi}{3}\right)$$

$$=\frac{\pi}{3}$$

Question 17:

Find the value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.

Solution:

Since, $\tan\theta = -\tan(\pi - \theta)$

Therefore,

$$\begin{aligned}
\tan^{-1}\left(\tan\frac{3\pi}{4}\right) &= \tan^{-1}\left[-\tan\left(-\frac{3\pi}{4}\right)\right] \\
&= \tan^{-1}\left[-\tan\left(\pi-\frac{\pi}{4}\right)\right] \\
&= \tan^{-1}\left[-\tan\left(\frac{\pi}{4}\right)\right] \\
&= \tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right] \\
&= \left(-\frac{\pi}{4}\right)
\end{aligned}$$

Question 18:

Find the value of $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$.

Solution:

Let $\sin^{-1}\frac{3}{5} = x \Rightarrow \sin x = \frac{3}{5}$

Then,

$$\Rightarrow \cos x = \sqrt{1-\sin^2 x} = \frac{4}{5}$$

$$\Rightarrow \sec x = \frac{5}{4}$$

Therefore,

$$\begin{aligned}
\tan x &= \sqrt{\sec^2 x - 1} \\
&= \sqrt{\frac{25}{16}-1} \\
&= \frac{3}{4}
\end{aligned}$$

$$x = \tan^{-1}\frac{3}{4}$$

$$\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4} \qquad \ldots(1)$$

Now,

$$\cot^{-1}\frac{3}{2}=\tan^{-1}\frac{2}{3} \qquad \ldots(2)$$

Thus, by using (1) and (2)

$$\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{2}{3}\right)=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)$$

$$=\tan\left[\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right]$$

$$=\tan\left(\tan^{-1}\frac{17}{6}\right)$$

$$=\frac{17}{6}$$

Question 19:

$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to

(A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Solution:

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)=\cos^{-1}\left(\cos\frac{-7\pi}{6}\right)$$

$$=\cos^{-1}\left[\cos\left(2\pi-\frac{7\pi}{6}\right)\right]$$

$$=\cos^{-1}\left[\cos\left(\frac{5\pi}{6}\right)\right]$$

$$=\frac{5\pi}{6}$$

Thus, the correct option is B.

Question 20:

$\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

Solution:

Let $\sin^{-1}\left(-\frac{1}{2}\right)=x$

Hence,

$$\sin x=-\frac{1}{2}$$
$$=-\sin\frac{\pi}{6}$$
$$=\sin\left(-\frac{\pi}{6}\right)$$
$$x=-\frac{\pi}{6}$$

Since, Range of principal value of $\sin^{-1}(x)=\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$.
Therefore,

$$\sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$$

Then,

$$\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)=\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)$$
$$=\sin\left(\frac{\pi}{2}\right)$$
$$=1$$

Thus, the correct option is D.

Question 21:

Find the values of $\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)$ is equal to

(A) π (B) $-\frac{\pi}{2}$ (C) 0 (D) $2\sqrt{3}$

Solution:

Let $\tan^{-1}\sqrt{3}=x$

Hence,

$$\tan x = \sqrt{3} = \tan\frac{\pi}{3}\text{, where } \frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Therefore, $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Now, let $\cot^{-1}\left(-\sqrt{3}\right) = y$

Hence,

$$\begin{aligned} \cot y &= \left(-\sqrt{3}\right) \\ &= -\cot\left(\frac{\pi}{6}\right) \\ &= \cot\left(\pi - \frac{\pi}{6}\right) \\ &= \cot\left(\frac{5\pi}{6}\right) \end{aligned}$$

Since, Range of principal value of $\cot^{-1} x = (0, \pi)$

Therefore,

$$\cot^{-1}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$$

Then,

$$\begin{aligned} \tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right) &= \frac{\pi}{3} - \frac{5\pi}{6} \\ &= -\frac{\pi}{2} \end{aligned}$$

Thus, the correct option is B.